On the Doctrine of Successive Lives. By PETER GRAY, ESQ., F.R.A.S.

PART II.*—APPLICATION.

(69.) THE chief, perhaps the sole, applications of the Theory delivered in Part I., are to Church Livings and Copyhold Estates. Of these applications we shall treat in order.

I.—CHURCH LIVINGS.

(70.) The principle on which the valuation of the rights of presentation to Church livings is subjected to treatment by this theory is, that the worth of the living is greater than that of the services to be performed by the holder of it. It is conceived, therefore, that the patron, in presenting a clerk to a benefice, is entitled to consider that, in addition to affording the clerk an opportunity for the exercise of his vocation, he is conferring upon him also an annuity due for his life, whose amount is the excess of the value of the benefice over that of the clerk's services. The law sanctions the arrangements that arise from this mode of viewing the subject, and permits the open purchase and sale of the right of presentation, whether for ever or for any number of times; save and except during the "mortal sickness" of an incumbent, or during the period between his death and the appointment of his successor; the purchase of a right of presentation in these circumstances being held to be simony, and the guilty party consequently being amenable to law.

(71.) Whether it be possible altogether to divest of a simoniacal character transactions in which a money value is put upon the *cure of souls,* and whether the state of things in which such transactions originate be a fit and proper one, are questions on which the writer confesses that he entertains strong opinions; and he has felt that, with those opinions, in giving forth the present brief series of papers, he exposes himself to the charge of showing how to commit simony on scientific principles. He cannot repel the charge: all he can say is, that although what he has done, and is about to do, may have the effect of modifying the details of some of those transactions, the morality of which he considers to be so questionable, his labours can hardly have the effect of increasing

* For Part I., see page 1 of this volume.

the number of such transactions. Having thus entered his caveat, he now proceeds to the consideration of the subject.

(72.) A few preliminary details are requisite. We have seen (38) that when the matter under consideration is a *perpetual advowson*, or right of presentation for ever, its value is independent of the ages of the successive nominees at appointment. In this case, therefore, the ages at nomination do not require to be, and, indeed, do not admit of being, taken account of. It is otherwise when the transaction has reference to a single presentation. The patron considers that, in granting a right to present, he is conferring a power of bestowing the benefice on the best life which the law allows, that is, age 24; and he will regulate his demands accordingly. Circumstances however may be such, that the purchaser will have no objection to come under an engagement that his nominee shall not be under a specified age, exceeding 24. In such case it will be no detriment to the interest of the patron if he so far relax his claim as to make the specified age, or even an age a year or two beyond it, the basis of his calculations. The greater the age of a nominee is at appointment, the sooner, in all likelihood, will the right of presentation have to be again exercised.

(73.) As already intimated, it is with the excess of the annual value of the living over that of the services to be performed, that we have here to deal. This excess we shall hereafter call the *net annual value:* how it is to be ascertained in each case it does not fall within our province to inquire. Sufficient information on the subject will be found in De Morgan's *Essay on Probabilities*, pp. 228, 229. It will be advantageous also to read the article "Advowson," in the *Penny Cyclopædia.*

(74.) The accompanying table, by which the following examples are worked, is derived from the Carlisle Table of Mortality, at 3 per cent. The writer does not take upon him to recommend these data as the most suitable to be employed in the solution of problems of the kind under consideration. They have been adopted for no other reason than that the abundance of existing deductions from the data in question rendered the requisite tables more easy of formation than they would have been if any other data had been made use of. In point of fact, the first two columns, A_x and log. A_x, are extracted from a work with the preparation of which the writer was connected; and the second column, log. $(1-A_x)$, was formed from certain logarithms tabulated in the same work. Thus (19),*

* The references up to (68) inclusive are to Part I.

TABLE.—*Carlisle,* 3 *per cent.*

Log. $(1+P)=1{\cdot}5357159$.

x	A_x	Log. A_x	Log. $(1-A_x)$	x	A_x	Log. A_x	Log. $(1-A_x)$
0	0·4664129	$\bar{1}$·6687706	$\bar{1}$·7272055	52	0·5759891	$\bar{1}$·7604143	$\bar{1}$·6273771
1	·3858945	·5864686	·7882431	53	·5869909	·7686313	·6159600
2	·3446464	·5373737	·8164756	54	·5981107	·7767816	·6041066
3	·3102056	·4916496	·8387199	55	·6094554	·7849419	·5916708
4	·2926733	·4663832	·8496202	56	·6209452	·7930533	·5787022
5	·2807962	·4483912	·8568520	57	·6325930	·8011244	·5651478
6	·2763374	·4414397	·8595362	58	·6441340	·8089762	·5512865
7	·2757316	·4404866	·8598998	59	·6551096	·8163140	·5376813
8	·2776499	·4434974	·8587478	60	·6652994	·8230171	·5246565
9	·2812506	4490935	·8565775	61	·6743529	·8288872	·5127474
10	·2860596	·4564565	·8536620	62	·6832484	·8345786	·5007189
11	·2914607	·4645800	·8503641	63	·6922331	·8402524	·4882223
12	·2968148	·4724856	·8470697	64	·7015856	·8460807	·4748198
13	·3022304	·4803382	·8437120	65	·7111452	·8519583	·4606797
14	·3077098	·4881413	·8402882	66	·7210172	·8579456	·4455778
15	·3131464	·4957474	·8368644	67	·7312245	·8640507	·4293904
16	·3183208	·5028650	·8335801	68	·7416953	·8702255	·4121327
17	·3233312	·5096475	·8303762	69	·7524471	·8764759	·3936680
18	·3283874	·5163864	·8271188	70	·7634013	·8827529	·3740121
19	·3335993	·5232251	·8237356	71	·7746662	·8891146	·3528267
20	·3389726	·5301646	·8202195	72	·7852693	·8950186	·3318941
21	·3445135	·5372062	·8165638	73	·7948508	·9002856	·3120699
22	·3503367	·5444856	·8126884	74	·8033334	·9048958	·2937311
23	·3563450	·5518706	·8086533	75	·8103313	·9086626	·2779961
24	·3625454	·5593624	·8044493	76	·8171777	·9123165	·2620299
25	·3689454	·5669621	·8000670	77	·8235205	·9156744	·2466941
26	·3754457	·5745471	·7955702	78	·8299583	·9190563	·2305555
27	·3821569	·5822417	·7908784	79	·8371339	·9227949	·2118311
28	·3888743	·5898092	·7861306	80	·8437460	·9262117	·1938316
29	·3952804	·5969052	·7815543	81	·8509109	·9298841	·1734455
30	·4012542	·6034195	·7772426	82	·8573503	·9331583	·1542711
31	·4073039	·6099186	·7728322	83	·8639269	·9364770	·1337733
32	·4135378	·6165152	·7682403	84	·8702706	·9396543	·1130390
33	·4200689	·6233205	·7633764	85	·8768187	·9429098	·0905451
34	·4269109	·6303372	·7582223	86	·8825335	·9457312	·0699138
35	·4339711	·6374608	·7528388	87	·8871842	·9480138	·0523704
36	·4412589	·6446935	·7472106	88	·8900203	·9493999	·0413129
37	·4486792	·6519359	·7414045	89	·8932555	·9509757	·0283458
38	·4562370	·6591905	·7354098	90	·8980959	·9533227	·0081919
39	·4639381	·6664600	·7292152	91	·8986240	·9535780	·0059361
40	·4715802	·6735556	·7229794	92	·8958153	·9522185	·0178029
41	·4789515	·6802915	·7168784	93	·8926248	·9506689	·0309030
42	·4862428	·6868532	·7107581	94	·8911951	·9499728	·0366485
43	·4935512	·6933322	·7045357	95	·8905746	·9496703	·0391182
44	·5010825	·6999092	·6980290	96	·8921201	·9504233	·0329418
45	·5088469	·7065871	·6912170	97	·8963512	·9524782	·0155650
46	·5169592	·7134563	·6839839	98	·9013108	·9548746	$\bar{2}$·9942707
47	·5254367	·7205204	·6762945	99	·9088092	·9584727	·9599522
48	·5344005	·7278668	·6680128	100	·9218675	·9646685	·8928330
49	·5440791	·7356621	·6588895	101	·9351016	·9708588	·8122355
50	·5543027	·7437470	·6490398	102	·9484163	·9769990	·7125133
51	·5650965	·7521226	·6383934	103	·9614478	·9829257	·5860489
				104	·9708739	·9871628	·4642841

$$1-A_x=\frac{1+a_x}{1+P}=\frac{N_{x-1}}{D_x}\div(1+P);$$

$$\therefore \text{log. }(1-A_x)=\text{log. }N_{x-1}+\text{colog. }D_x+\text{colog. }(1+P).$$

This method of formation is easier than the more obvious one, while the results, especially as regard the older ages, are more correct than they would have been had any other method been employed.

(75.) *Example* 1.—Required the value of the advowson of a living of the net annual value of £317. 10*s*., the age of the incumbent being 56.

By (38) the required value is

$$(1+P)\,A_{56}\times 317{\cdot}5$$

$1+P$,	log.	$1{\cdot}5357159$
A^{56},	„	$\bar{1}{\cdot}7930533$
$317{\cdot}5$,	„	$2{\cdot}5017437$
$6768{\cdot}820$,	„	$3{\cdot}8305129$ ∴ £6768. 16*s*. 5*d*. = value required.

(76.) *Ex*. 2.—Required the value of the 1st, 2nd, 3rd, and 4th presentations to the same living, the age of the incumbent being as above, and the common age of the nominees at appointment 24.

The formula here requisite is that of (34), namely—

$$(1+P)\,(1-A_y)\,A_x\,.\,A_y^{\,n-1},$$

in which $x=56$, $y=24$, and $n=1, 2, 3, 4$, successively.

$1+P$,	log.	$1{\cdot}5357159$						
$1-A_{24}$,	„	$\bar{1}{\cdot}8044493$						
A_{56},	„	$\bar{1}{\cdot}7930533$						
$A_{24}^{\,0}=1$,	„	$0{\cdot}0000000$						
$317{\cdot}5$,	„	$2{\cdot}5017437$						
$4314{\cdot}815$,	„	$3{\cdot}6349622$ ∴	£4314	16	4 =	value of	first presentation.	
A_{24},	„	$\bar{1}{\cdot}5593624$						
$1564{\cdot}316$,	„	$3{\cdot}1943246$ ∴	£1564	6	4 =	„	second	„
A_{24},	„	$\bar{1}{\cdot}5593624$						
$567{\cdot}136$,	„	$2{\cdot}7536870$ ∴	£567	2	9 =	„	third	„
A_{24},	„	$\bar{1}{\cdot}5593624$						
$205{\cdot}612$,	„	$2{\cdot}3130494$ ∴	£205	12	3 =	„	fourth	„
			£6,651	17	8 =	„	first four	„

(77.) It is here shown that when the value of the first presentation is found, the succeeding values are formed in order by the introduction of the factor A_{24}; and the sum of the values thus obtained is the aggregate value of the whole. If, however, the value of a remote presentation only is wanted, it may be obtained without going through all the foregoing process.

(78.) *Ex.* 3.—Required the value of the fourth presentation to the above living.

The formula for this case is still that of (34), when $x=56$, $y=24$, and $n=4$.

$$(1+P)(1-A_{24})A_{56}\,.\,A_{24}^{3}.$$

A_{24},	log.	$\bar{1}\cdot5593624$
		3
A_{24}^{3},	„	$\bar{2}\cdot6780872$
$1+P$,	„	$1\cdot5357159$
$1-A_{24}$,	„	$\bar{1}\cdot8044493$
A_{56},	„	$\bar{1}\cdot7930533$
317·5,	„	2·5017437
205·612,	„	2·3130494 ∴ £205. 12*s.* 3*d.* = value required.

(79.) *Ex.* 4.—Required the value of the first four presentations to the foregoing living.

Here the formula is that of (40), when $x=56$, $y=24$, and $n=4$.

$$(1+P)(1-A_{24}^{4})A_{56}.$$

A_{24},	log.	$\bar{1}\cdot5593624$
		4
$A_{24}^{4}=\cdot0172763$,	„	$\bar{2}\cdot2374496$
$1-A_{24}^{4}=\cdot9827237$,	„	$\bar{1}\cdot9924314$
$1+P$,	„	$1\cdot5357159$
A_{56},	„	$\bar{1}\cdot7930533$
317·5,	„	2·5017437
6651·878,	„	3·8229443 ∴ £6,651. 17*s.* 7*d.* = value required.

(80.) *Ex.* 5.—Required the value of the fifth and following presentations for ever.

The formula here is that of (43), when $x=56$, $y=24$, and $n=5$.

$$(1+P)A_{56}\,.\,A_{24}^{4}.$$

$1+P$,	log.	$1\cdot5357159$	
A_{56},	„	$\bar{1}\cdot7930533$	
A_{24}^{4},	„	$\bar{2}\cdot2374496$	*Ex.* 4.
$317\cdot5$,	„	$2\cdot5017437$	
$116\cdot940$,	„	$2\cdot0679625$	$\therefore$ £116. 18*s.* 10*d.* = value required.

The sum of this result and that of Example 4, is £6768. 16*s.* 5*d.*, the value of the advowson, as already found in Example 1.

(81.) *Ex.* 6.—The foregoing advowson is held by two tenants in common (and their heirs), who present alternately. Required the value of their several interests.

The formula here is that of (51), which, when $m=1$ and $t=2$, will denote the value of the interest of the first tenant; and when $m=2$ and $t=2$, will denote the value of the interest of the second tenant.

We thus have for the interest of the first tenant (that is, of the party who has the first turn)—

$$\frac{(1+P)(1-A_{24})\,A_{56}}{1-A_{24}^{2}};$$

and for the interest of the second—

$$\frac{(1+P)(1-A_{24})\,A_{56}\,.\,A_{24}}{1-A_{24}^{2}};$$

which last expression we see differs from the former only in having in the numerator the additional factor A_{24}. We may therefore avail ourself of this relation, to deduce the numerical value of the one expression from that of the other.

A_{24},	log.	$\bar{1}\cdot5593624$	
		2	
A_{24}^{2}, $\cdot1314392$,	„	$\bar{1}\cdot1187248$	
$1-A_{24}^{2}$, $\cdot8685608$,	„	$\bar{1}\cdot9388002$	
	colog.	$0\cdot0611998$	
$1+P$,	log.	$1\cdot5357159$	
$1-A_{24}$,	„	$\bar{1}\cdot8044493$	
A_{56},	„	$\bar{1}\cdot7930533$	
$317\cdot5$,	„	$2\cdot5017437$	
$4967\cdot776$,	„	$3\cdot6961620$	$\therefore$ £4967 15 6 = interest of first tenant.
A_{24},	„	$\bar{1}\cdot5593624$	
$1801\cdot045$,	„	$3\cdot2555244$	$\therefore$ £1801 0 11 = „ second do.
			£6768 16 5 = value of advowson as in Ex. 1.

The problem to which this case belongs is so important, that it may be well to give another example of the application of the foregoing formula.

(82.) *Ex.* 7.—Determine the value of the interest of each tenant in common when their number is three.

Here $t=3$, and $m=1$, 2, 3 successively. We thus have for the interests of the several tenants, respectively—

$$\text{Of the first} \quad . \quad . \quad \frac{(1+\mathrm{P})(1-\mathrm{A}_{24})\,\mathrm{A}_{56}}{1-\mathrm{A}_{24}^{\;3}};$$

$$\text{Of the second} \quad . \quad \frac{(1+\mathrm{P})(1-\mathrm{A}_{24})\,\mathrm{A}_{56}\,.\,\mathrm{A}_{24}}{1-\mathrm{A}_{24}^{\;3}};$$

$$\text{Of the third} \quad . \quad . \quad \frac{(1+\mathrm{P})(1-\mathrm{A}_{24})\,\mathrm{A}_{56}\,.\,\mathrm{A}_{24}^{\;2}}{1-\mathrm{A}_{24}^{\;3}}.$$

Here the same thing holds as in the preceding example. Each value, after the first, is derived from that which precedes it by multiplication by the factor A_{24}.

A_{24},	log.	$\bar{1}\cdot5593624$		
		3		
$\mathrm{A}_{24}^{\;3}=\cdot0476527$,	„	$\bar{2}\cdot6780872$		
$1-\mathrm{A}_{24}^{\;3}=\cdot9523473$,	„	$\bar{1}\cdot9787953$		
	colog.	0·0212047		
$1+\mathrm{P}$,	log.	1·5357159		
$1-\mathrm{A}_{24}$,	„	$\bar{1}\cdot8044493$		
A_{56},	„	$\bar{1}\cdot7930533$		
317·5	„	2·5017437		
4530·716,	„	3·6561669	∴ £4530 14 4	= interest of first tenant.
A_{24},	„	$\bar{1}\cdot5593624$		
1642·590,	„	3·2155293	∴ £1642 11 10	= „ second do.
A_{24},	„	$\bar{1}\cdot5593624$		
595·514,	„	2·7748917	∴ £595 10 3	= „ third do.
			£6768 16 5	= value of advowson.

(83.) *Ex.* 8.—Determine the interest of the third tenant in common, irrespective of those of the first and second.

The formula is still that of (51), when $x=56$, $y=24$, $m=2$, and $t=3$.

$$\frac{(1+P)(1-A_{24})A_{56}\,.\,A_{24}^{2}}{1-A_{24}^{3}}.$$

$1+P$,	log.	$1 \cdot 5357159$
$1-A_{24}$,	,,	$\bar{1} \cdot 8044493$
A_{56},	,,	$\bar{1} \cdot 7930533$
A_{24},	,,	$\bar{1} \cdot 5593624$
,,	,,	$\bar{1} \cdot 5593624$
$317 \cdot 5$,	,,	$2 \cdot 5017437$
$1-A_{24}^{3}$,	colog.	$0 \cdot 0212047$ *Ex.* 7
$595 \cdot 514$,	log.	$2 \cdot 7748917$ ∴ £595. 10*s.* 3*d.* = interest of third tenant.

(84.) It has not been thought necessary to give examples of the application of all the formulæ deduced in Part I. In particular, none have been given of the formulæ in which the succeeding lives are supposed to be of different ages at nomination, as cases in which it is requisite to make that supposition will hardly occur in practice. It is believed that enough has been done to obviate all difficulty in the treatment of any case that may arise.

II.—Copyhold Estates.

(85.) The feature in copyhold tenures with which we have to do is, that they are held of the lord of the manor, subject to the payment to him not of an annual sum in name of rent, but of a fixed sum at the death of the single tenant, if there be but one, or of each tenant, if there are more than one—a new life being nominated to take the place of each that fails, at the end of the year in which such failure takes place. It thus appears, that the greater the number of lives on which a copyhold tenure depends, the greater is the value of the fines to the receiver. In point of fact, a lease held on three lives, for example, will pay just as many fines to the lord as three leases held each on one life, the lives in both cases being of the same ages respectively.

(86.) The right of nominating new lives, as the lives in possession drop, rests with the copyholders and their heirs; and as it is obviously their interest that the fines to be paid should be as few as possible, it may be fairly assumed, that the life *put in* at each renewal will be the best that the tables afford. As it is not necessary that the lives nominated should have an interest in the lease, there can be no difficulty in the fulfilment of this condition. The best life by the Carlisle Table is age 7.

(87.) It will readily appear that, in questions relating to the

value of the fines, that of the estate in respect of which they are paid does not enter into the computation.

(88.) *Ex.* 1. — A copyhold estate is held on a single life, subject to a renewal fine, at the decease of each tenant, of £1520. The life now in the lease is 37 years of age, and it is required to determine the present value of the first, second, third, &c. fines.

The formula here is that of (61), when $x=37$, $y=7$ (the renewals being always made with lives of 7 years of age), and $n=1, 2, 3$, &c. We thus have for the value (if the fine is £1)

Of the first fine, A_{37},
„ second „ $A_{37} . A_7$,
„ third „ $A_{37} . A_7^2$,

and so on; so that here, as in previous cases, the value of each fine after the first is derived from that of the preceding, by the introduction as a factor of A_7.

1520,	log.	3·1818436		
A_{37},	„	$\bar{1}$·6519359		
681·992,	„	2·8337795	∴ £681 19 10 =	value of first fine.
A_7,	„	$\bar{1}$·4404866		
188·047,	„	2·2742661	∴ £188 0 11 =	„ second fine.
A_7,	„	$\bar{1}$·4404866		
51·850,	„	1·7147527	∴ £51 17 0 =	„ third fine.
			£921 17 9 =	„ first three fines.

(89.) *Ex.* 2.—Required the value of the third fine.
The formula is (61),

$$A_{37} . A_7^2.$$

A_7,	log.	$\bar{1}$·4404866
		2
A_7^2,	„	$\bar{2}$·8809732
A_{37},	„	$\bar{1}$·6519359
1520,	„	3·1818436
51·850,	„	1·7147527 ∴ £51. 17*s*. = value required.

(90.) *Ex.* 3.—Required the value of the first three fines.
The formula here is that of (65), when $x=37$, $y=7$, and $n=3$.

$$\frac{A_{37}\,(1-A_7^3)}{1-A_7}.$$

$$
\begin{array}{rrl}
A_7, & \text{log.} & \bar{1}\cdot4404866 \\
 & & \qquad\quad 3 \\
\hline
A_7^3 = \cdot0209633, & ,, & \bar{2}\cdot3214598 \\
\hline
1-A_7^3 = \cdot9790367, & ,, & \bar{1}\cdot9907989 \\
A_{37}, & ,, & \bar{1}\cdot6519359 \\
1-A_7, & \text{colog.} & 0\cdot1401002 \\
1520, & \text{log.} & 3\cdot1818436 \\
\hline
921\cdot889, & ,, & 2\cdot9646786 \; \therefore \text{£}921.\ 17s.\ 9d. = \text{value required.}
\end{array}
$$

Here colog. $(1-A_7)$ being the arithmetical complement of log. $(1-A_7)$, it is taken from the column log. $(1-A_x)$ almost as easily as log. $(1-A_7)$ itself.

(91.) *Ex.* 4.—Required the value of all the fines for ever.

The formula here is that of (66), where $x = 37$, and $y = 7$.

$$\frac{A_{37}}{1-A_7}.$$

$$
\begin{array}{rrl}
1520, & \text{log.} & 3\cdot1818436 \\
A_{37}, & ,, & \bar{1}\cdot6519359 \\
1-A_7, & \text{colog.} & 0\cdot1401002 \\
\hline
941\cdot629 & \text{log.} & 2\cdot9738797 \; \therefore \text{£}941.\ 12s.\ 7d. = \text{value required.}
\end{array}
$$

(92.) This is the sum which the copyholder might now equitably pay for the conversion of his copyhold into a freehold; and if it be divided by the value of a perpetuity of £1, *i.e.* by $1 \div r$, or, which is the same thing, multiplied by r (viz., ·03), we obtain $28\cdot249 =$ £28. 5*s.*, which is the perpetual annual rent for which the fines might be commuted. It is also the *net* annual premium, in consideration of which an office might undertake to pay the fines as they become due.

(93.) *Ex.* 5.—A copyhold estate is held on three lives, each renewable at the end of the year in which it drops, by a life of 7 years of age, on payment of a fine of £3000; the lives at present in the lease are aged respectively 30, 42, and 65 years. It is required to determine the present value of all the fines for ever, and also the equivalent annual payment for which they might be commuted.

By (66) the value of the fines in respect of (30) and his successors is

$$\frac{3000\,A_{30}}{1-A_7};$$

the like in respect of (42) and his successors is

$$\frac{3000\,A_{42}}{1-A_7};$$

and the like in respect of (65) and his successors is

$$\frac{3000\,A_{65}}{1-A_7};$$

hence the total value required is

$$\frac{3000\,(A_{30}+A_{42}+A_{65})}{1-A_7}.$$

$A_{30}=\cdot4012542$
$A_{42}=\cdot4862428$
$A_{65}=\cdot7111452$

1·5986422,	log.	0·2037513
3000,	„	3·4771213
$1-A_7$,	colog.	0·1401002
6621·750,	log.	3·8209728 ∴ £6621. 15*s.* = value required.
·03		

198·65250, ∴ £198. 13*s.* = annual payment required.

London, 13*th January,* 1852.